INTEGRATION WITH SIMPLE SUBSTITUTION

We use simple substitution to integrate functions that are complex to integrate, and functions that do not fit forms such as $\int \frac{f'(x)}{f(x)}\ dx$, $\int \frac{ax+b}{cx+d}\ dx$, $\int f'.g'\ dx$ etc.

We tend to use simple substitution with composite functions, or products of composite functions, and replace the inner function with a variable, usually $'u'$.

Example: Integrate these functions using substitution.

1) $\int (2x-7)^5\ dx$

Use $u = 2x - 7$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}du$

$\int (2x-7)^5\ dx = \int (u)^5\ \frac{1}{2}du = \frac{1}{2}\int u^5\ du = \frac{1}{2}\cdot\frac{u^6}{6} + c = \frac{1}{12}u^6 + c$

Substitute $u = 2x - 7$ back into result, therefore:

$\int (2x-7)^5\ dx = \frac{1}{12}(2x-7)^6 + c$

2) $\int x(x-4)^3\ dx$

Use $u = x - 4$

$\frac{du}{dx} = 1$

$dx = du$

Also: $x = u + 4$

$\int x(x-4)^3\ dx = \int (u+4)(u)^3\ du = \int u^4 + 4u^3\ du$

$= \frac{1}{5}u^5 + \frac{4}{4}u^4 + c$

Substitute $u = x - 4$ back into the result, therefore:

$\int x(x-4)^3\ dx = \frac{1}{5}(x-4)^5 + (x-4)^4 + c$

3) $\int 8x(2x^2-3)^4\ dx$

Use $u = 2x^2 - 3$

$\frac{du}{dx} = 4x$

$dx = \frac{1}{4x}du$

$\int 8x(2x^2-3)^4\ dx = \int 8x(u)^4\ \frac{1}{4x}du = \int \frac{8x}{4x}u^4\ du = \int 2u^4\ du$

$= \frac{2}{5}u^5 + c$

Substitute $u = 2x^2 - 3$ back into the result, therefore:

$\int 8x(2x^2-3)^4\ dx = \frac{2}{5}(2x^2-3)^5 + c$

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